Layered synchronous propagation of noise-induced chaotic spikes in linear arrays

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Stable propagation of noise-induced synchronous spiking in uncoupled linear neuron arrays is studied numerically. The chaotic neurons in the unidirectionally coupled linear array are modeled by Hindmarsh-Rose neurons. Stability analysis shows that the synchronous chaotic spiking can be successfully transmitted to cortical areas through layered synchronization in the neural network under certain conditions of the network structure.

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Studies of cooperative behaviors in coupled chaotic oscillators are frequently based on the analysis of the phenomenon of chaos synchronization [1]. Different types of synchrony between chaotic oscillators along with the various mechanism responsible for the onset of such synchronization have been studied [2]. Recently much attention has been focused on the synchronization and rhythmic processes in neurobiology and physiology [3], e.g., bursts in different neurons can be synchronized, as analyzed theoretically [4] and experimentally [5], if the neurons are either coupled [6] or driven by a common noise [7]. A model of dynamical encoding and propagating by neural networks with feed-forward arrangements has been introduced [8]; however, the core issue of the whole process of signal transmission especially the cooperative transmission in neural system has not been addressed thus far.

With reference to this issue, we introduce a model built upon simplicity requirements; namely, we consider linear and unidirectional interneuron coupling; furthermore, we take just nearest-neighbor coupling, avoiding architecturally complicated connections, and those outermost neurons receive a common Gaussian noise as described in Fig. 1. Under these assumptions we address the phenomenon that signal propagates synchronously layer by layer along the linear arrays of chaotic neurons.

To introduce the method and illustrate its possible application in neuroscience, we consider a realistic model of cooperative activity in a population of neurons, where individual neuron is described by the Hindmarsh-Rose equations [9]: $\dot{x}=y-ax^3+bx^2-z+I, \dot{y}=c-dx^2-y, \dot{z}=r[s(x-x^0)-z]$, where x is the membrane potential, y is associated with the fast current, Na^+ or K^+, and z with the slow current, for example, Ca^{2+}. Here $a=1.0$, $b=3.0$, $c=1.0$, $d=5.0$, $s=4.0$, $r=0.006$, $x^0=-1.60$, and I is the constant stimulus. x, y are fast variables and z is a slow variable. r is the ratio of fast/slow time scales. This system can exhibits different kinds of dynamic behaviors: For sufficiently low values of $I(0<I<I^{(1)}\approx 1.32)$ the neuron is in a stable quiescent state. As I increases from $I^{(1)}$ to $I^{(2)}\approx 2.92$, there exists a bifurcation to a low-frequency repetitive firing state consisting of a train of regularly spaced spikes. Further increase of $I(I^{(2)}<I<I^{(3)}\approx 3.40)$ leads to a multi-time-scale spike-burst chaotic behavior. The system is finally in a high-frequency repetitive firing state for $I>I^{(3)}$ [9]. Now we consider multiple unidirectionally coupled linear arrays of Hindmarsh-Rose neurons in the presence of a common noise applied to the outermost neurons. The motion can be described by the following equations:

$$\dot{x}_{ij}=y_{ij}-a_{ij}x_{ij}^3+b_{ij}x_{ij}^2-z_{ij}+I_{ij}^{ext},$$

$$\dot{y}_{ij}=c_{ij}-d_{ij}x_{ij}^2-y_{ij},$$

$$\dot{z}_{ij}=r_{ij}[s_{ij}(x_{ij}-x_{ij}^0)-z_{ij}]. \quad (1)$$

Here $i=1,\ldots,M$ is the ith linear array index, and $j=1,\ldots,N$ is the jth neuron index in the linear arrays. $(a_{ij},b_{ij},c_{ij},d_{ij},s_{ij},r_{ij},x_{ij}^0)$ are parameters of individual neuron. $I_{ij}^{ext}=I+D\xi$ for $j=1$ (the 1st layer), and $I_{ij}^{ext}=\epsilon(x_{ij-1}-X)$ for $j\geqslant 2$ (the jth layer). The noise ξ is a Gaussian one with $\langle\xi(t)\xi(t-\tau)\rangle=\delta(\tau)$, and D denotes the noise intensity; ϵ is the coupling strength, and X is the dynamic parameter for stable propagation of the spikes. It should be noted that $(a_{ij},b_{ij},c_{ij},d_{ij},s_{ij},r_{ij},x_{ij}^0)=(a_{i'j},b_{i'j},c_{i'j},d_{i'j},s_{i'j},r_{i'j},x_{i'j}^0)$, i.e., those neurons belong to the same layer are identical, otherwise, may be different. In our case, for simplicity, we choose $(a_{ij},b_{ij},c_{ij},d_{ij},s_{ij},r_{ij},x_{ij}^0)=(a,b,c,d,s,r,x^0)$, but we can get the similar conclusion if those neurons belong to different layers are different from each other. Because all the linear arrays are identical and do not interact with one another, we can study the layered synchronization of the neural

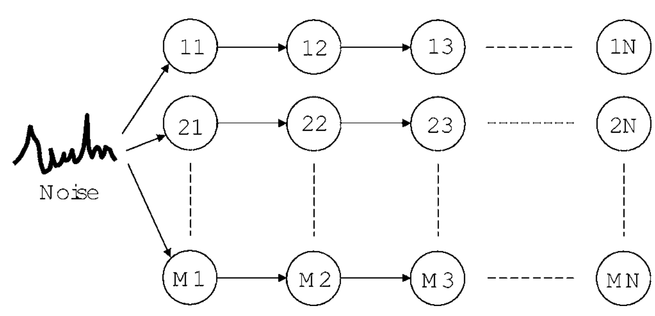

FIG. 1. Architecture of the model neural network.

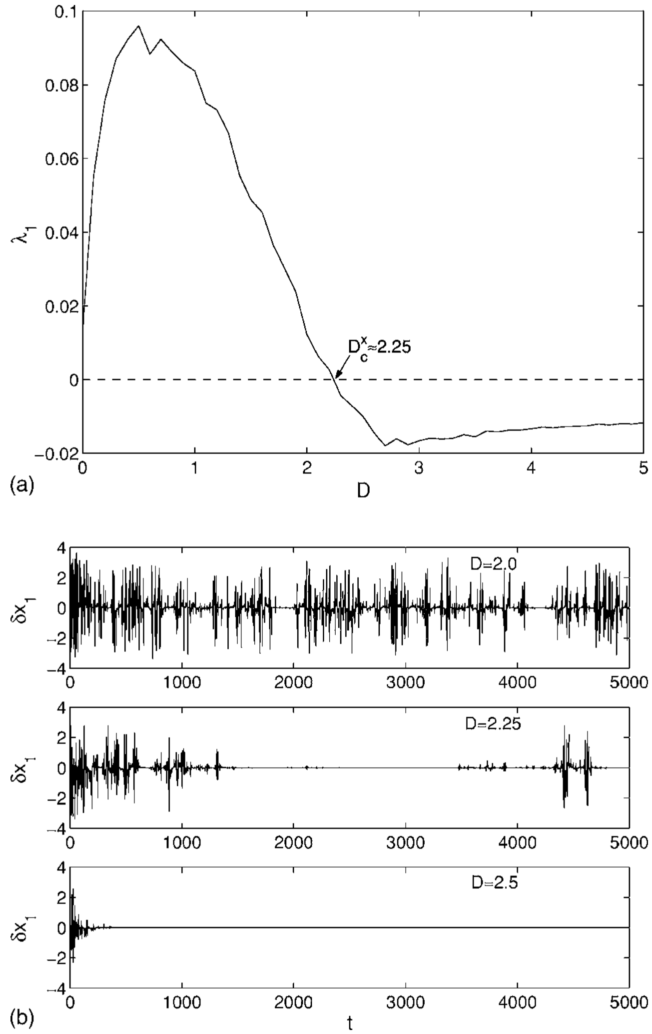

FIG. 2. (a) λ_1 versus noise intensity D for $I=3.2$. (b) Time series of $\delta x_1 = x_{i1} - x_{i'1}$ for different D. Other parameters are given in the text.

network in any two (i and i') of the M linear arrays. The initial conditions of the membrane potential and ion currents of each neuron are chosen randomly and independently. The equations were integrated using the stochastic Euler method with a time step of $\Delta t = 0.01$.

It has previously been observed that noise-induced complete synchronization (CS) can be realized in two uncoupled Hindmarsh-Rose neurons when a common Gaussian noise is applied to the equation of motion of the x variable [10]. In order for the stable propagation of the noise-induced synchronization in our model, two points should be stressed: First, the largest Lyapunov exponent (LLE) λ_1 of the difference equations $\dot{\delta}_{ij} = (\dot{\delta}^x_{ij}, \dot{\delta}^y_{ij}, \dot{\delta}^z_{ij}) = [\dot{x}_{ij} - \dot{x}_{i'j}, \dot{y}_{ij} - \dot{y}_{i'j}, \dot{z}_{ij} - \dot{z}_{i'j}] = [\delta^y_{ij} - 3ax_j^2\delta^x_{ij} + 2bx_j\delta^x_{ij} - \delta^z_{ij} + \epsilon\delta^x_{ij-1}, -2dx_j\delta^x_{ij} - \delta^y_{ij}, r(s\delta^x_{ij} - \delta^z_{ij})]$ should be negative, i.e., $\delta_{ij}(t) = (0,0,0)$ for $t \to \infty$, which gives the layered chaotic synchronization in the jth layer. Here (x_j, y_j, z_j) is the jth layer synchronization manifold, and $\epsilon\delta^x_{ij-1} = 0$ for $j=1$. Second, the parameter X should be carefully adjusted in order for the lossless and stable propagation of information. Now we discuss the stable propagation of noise-induced chaotic spikes by calculating LLE λ_1. Figure

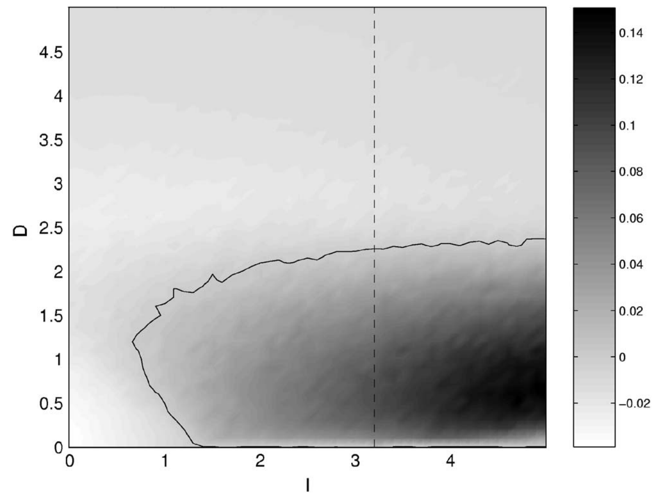

FIG. 3. λ_1 in (D-I) parameter space. The dashed line is drawn at $I=3.2$, and solid line is drawn at $\lambda_1 = 0$. Other parameters are the same as that in Fig. 2.

2(a) shows that, for $I=3.2$, the critical noise intensity $D_c^x \approx 2.25$ beyond which the LLE λ_1 becomes negative, and the chaotic neurons in the 1st layer are in complete synchronous state [see Fig. 2(b)]. Figure 3 exhibits the LLE λ_1 in (D-I) parameter space, which shows that when the noise intensity $D > D_c \approx 2.5$, the LLE λ_1 becomes negative for any value of I, i.e., the neurons driven by a common noise are in complete synchronized states. But D_c^x is different for different I. There is a common characteristic for different values of I, as seen in Fig. 2(a), with increasing the noise intensity, the LLE λ_1 increases from the value of isolated chaotic neuron, after reaching a peak, then decreases till to the negative value. So there is a range of noise intensity in which the value of LLE λ_1 is larger than the value of isolated chaotic neuron, i.e., the increase of noise intensity may not always increase the degree of synchronization. This phenomenon is similar to that in Ref. [11].

The synchronous chaotic spikes induced by noise transmit to the 2nd layer according to Eq. (1) with $I_{ij}^{ext} = \epsilon(x_{ij-1} - X)$ ($j=2$). To ensure the synchronization between neurons of the 2nd layer, we compute the LLE λ_1 for $X=-2.64$ and different ϵ. We fix the noise intensity $D=3.0$ to ensure the synchronization of neurons of the 1st layer. Figure 4(a) shows that the LLE λ_1 becomes negative when the coupling strength $0 < \epsilon < \epsilon_c^1 \approx 0.52$ and $\epsilon > \epsilon_c^2 \approx 3.70$, i.e., the neurons of the 2nd layer are in synchronized states. But in the former range, the coupling strength ϵ is too small to excite large numbers of spikes, while in the latter range, the ϵ is large enough to excite many spikes and ensure the synchronization between the neurons of the 2nd layer simultaneously [see Fig. 4(b)]. To explore the parameter range with negative LLE, we compute the λ_1 in the (X-ϵ) parameter space. Figure 5 shows that there are two separate regions with negative LLE. One region corresponds to the smaller ϵ, and the other to the larger ϵ. The latter parameter range makes sense for us because it ensures synchronous propagation of many spikes from layer to layer in our neural network.

In neurobiology, the outermost neurons which driven by

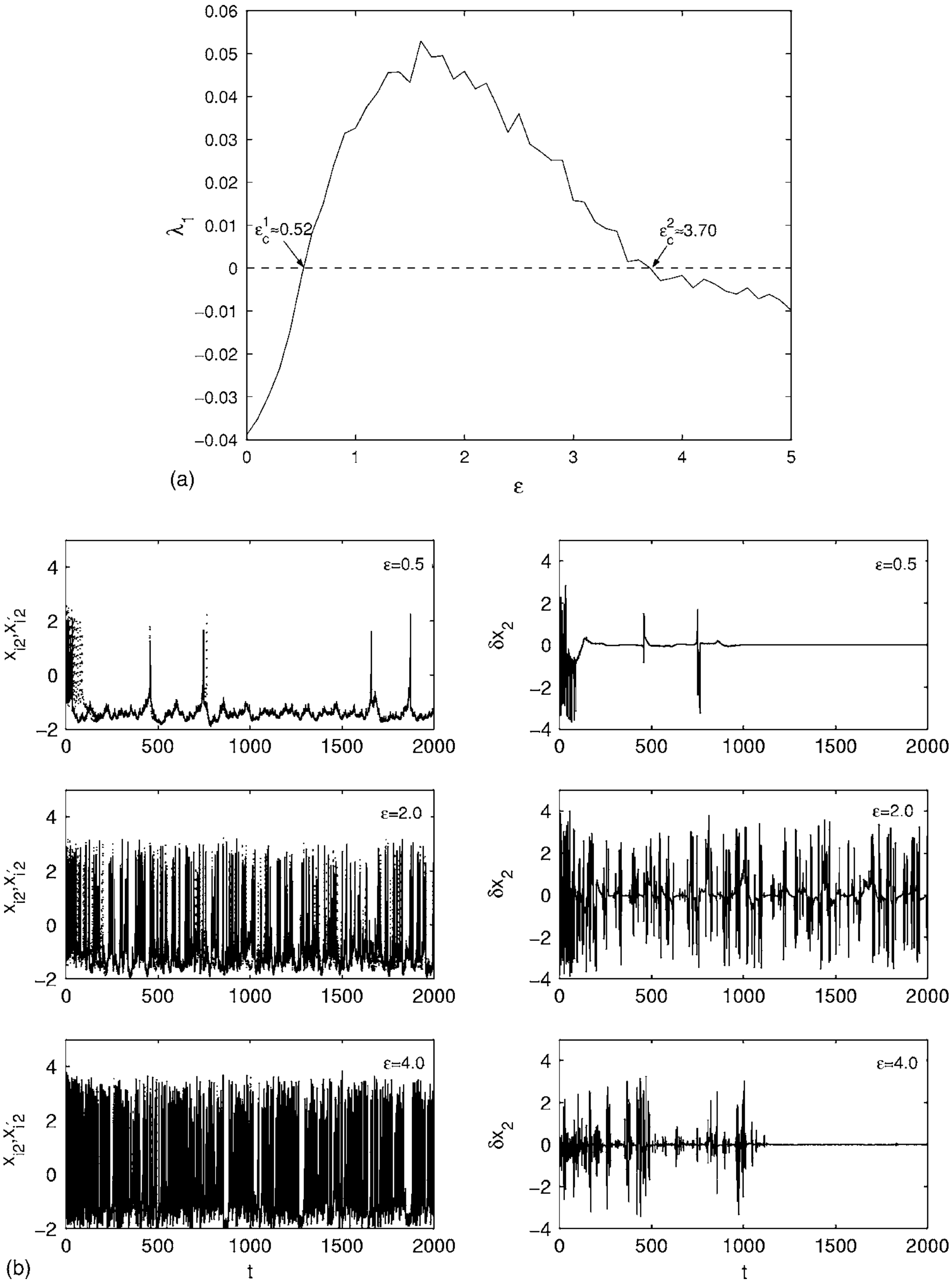

FIG. 4. (a) λ_1 versus coupling strength ϵ for $X=-2.64$. (b) Time series of x_{i2} (solid line), $x_{i'2}$ (dotted line), and $\delta x_2 = x_{i2} - x_{i'2}$ for different ϵ. The intensity of noise $D=3.0$ and other parameters are the same as that in Fig. 2.

external stimulus can be regarded as sensory neurons. When the sensory neurons (neurons of the 1st layer) receive the stimulus (a common Gaussian noise in our case), they will generate spikes which transmit to the inner neurons (neurons of the 2nd layer), then propagate to the next layer, and so on, until these spikes reach the cerebral cortex. Figure 6(a) shows the temporal behaviors of the difference motion $\delta x_j = x_{ij} - x_{i'j}$ (corresponding to fast dynamics) and $\delta z_j = z_{ij} - z_{i'j}$ (corresponding to slow dynamics) for $j = 20, 60, 100$, respectively. After the transient time interval $t_u := \delta x_j\ (\delta z_j) \neq 0$ for $t < t_u$ and $\delta x_j(\delta z_j) \approx 0$ for $t > t_u$, the two neurons of the same layer are in CS state. But the transient time interval t_u increases linearly with increasing layer index j [12]. It should be noted that the amplitude of δz_j is much smaller than that of δx_j [about 1:10 in our case; see Fig. 6(a)]. We have computed the finite time LLE λ_1 for different layers. The numerical results are shown in Fig. 6(b), from which we can see that λ_1 increases with increasing layer index j. In the computation we have chosen the finite time $t=6000$ to exhibit the different transient time intervals t_u for different layers. In the thermodynamic limit, i.e., $j \to \infty$, the layered CS will be maintained under certain conditions as discussed by Aranson *et al.* in Ref. [13].

Another characteristic for the stable propagation of the chaotic spikes with layered synchronization is its sensitivity to the symmetry of the dynamic parameters between the linear arrays. By means of the modification of the auxiliary system approach [14], we study the influence of asymmetry of the coupling constant ϵ on the layered synchronization (we also studied other kinds of asymmetry, such as r, and got the similar conclusion, here not shown). Taking $\epsilon = 5.0$ in the whole neural network, and $\epsilon' = \epsilon(1.0 + \Delta_{j_0})$ between the j_0th

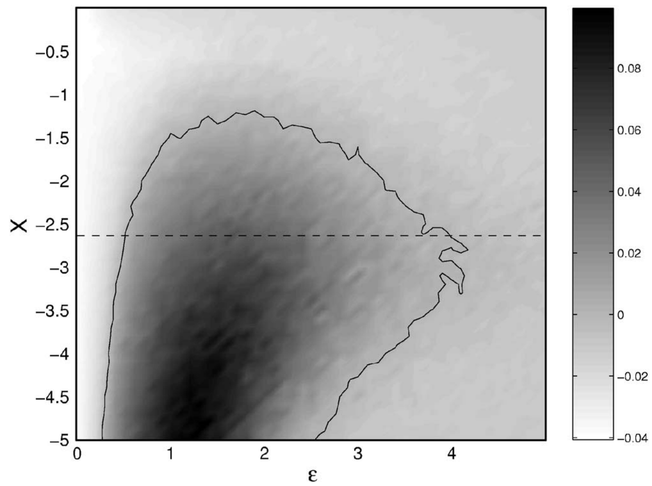

FIG. 5. λ_1 in $(X\text{-}\epsilon)$ parameter space. The dashed line is drawn at $X=-2.64$, and solid line is drawn at $\lambda_1=0$. Other parameters are the same to that in Fig. 4.

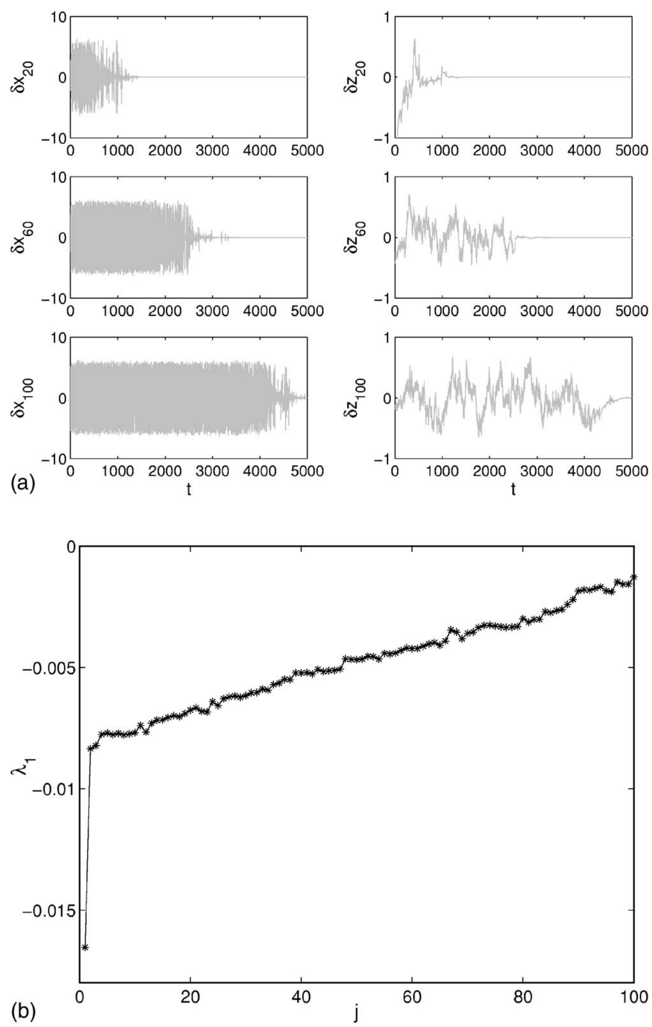

FIG. 6. (a) Temporal behaviors of the difference motion $\delta x_j = x_{ij} - x_{i'j}$ and $\delta z_j = z_{ij} - z_{i'j}$ for $j=20,60,100$, respectively. (b) The diagram of the finite time LLE λ_1 vs the layer index j in the systems. Here we choose the finite time $t=6000$, and other parameters are as follows: $D=3.0$, $I=3.2$, $X=-2.64$, and $\epsilon=5.0$.

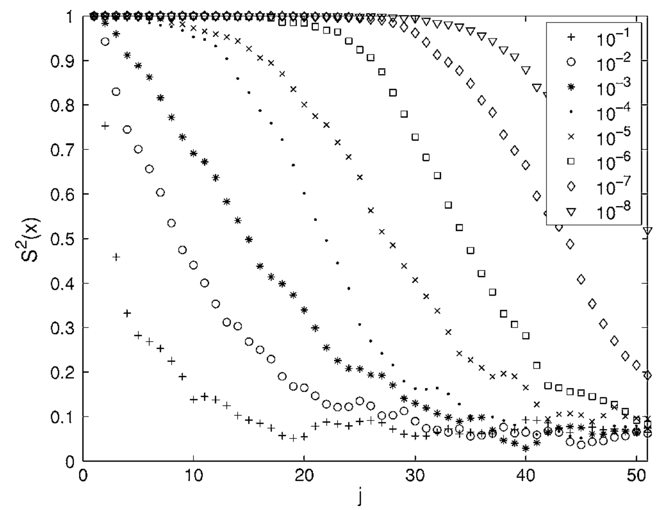

FIG. 7. The cross-correlation $S^2(x)$ along the linear array for different relative error Δ_1. Parameters are the same as those in Fig. 6.

neuron and the (j_0+1)th neuron in one (i or i') linear array, we compute the cross-correlation function

$$S^2(x) = \frac{\langle x_{ij}(t) x_{i'j}(t)\rangle}{[\langle x_{ij}^2(t)\rangle\langle x_{i'j}^2(t)\rangle]^{1/2}} \quad (j \geqslant j_0) \tag{2}$$

for different relative error Δ_1 (here $j_0=1$) of the coupling constant (see Fig. 7). The cross-correlation function $S^2(x)=1.0$ corresponds to the complete layered synchronization, and $S^2(x)=0$ corresponds to the complete unsynchronization of the same layer neurons. Figure 7 shows the $S^2(x)$ decreases along the linear array till to $S^2(x)\approx 0.1$ for $\Delta_1 \neq 0$ and $j\to\infty$. This residual correlation $S^2(x)\approx 0.1$ does not come from the correlation but the pulse width d of spikes. If $d=0$ then $S^2(x)=0$ for $\Delta_1\neq 0$ and $j\to\infty$. $S^2(x)$ decreases more rapidly for larger relative error Δ_1. We also calculated the cross-correlation function $S^2(z)$ and found that $S^2(z)$ is approximately 1.0 for different relative error Δ_1. Thus the successful synchronous propagation of signal (especially for fast variable, e.g., x,y) in the neural network means the symmetry of the dynamic structure of the network. Another point to be stressed is that the local noise has the similar role played by Δ_{j_0} and can destroy the layered CS.

In conclusion, we have introduced the neural network model as multiple linear arrays of coupling chaotic neurons which assume a collective state in the presence of a localized common noise. In this neural network, under certain conditions, the chaotic spikes induced by external noise can propagate stably and synchronously to the cerebral cortex via layered synchronization along the linear arrays, so that the precise spike timing is maintained.

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